## EELE 461/561 – Digital System Design

### Module #2 – Interconnect Modeling with Lumped Elements

- **Topics**

  1. Modeling Techniques
  2. Impedance of Resistors, Capacitors and Inductors

- **Textbook Reading Assignments**

  1. 3.1-3.7

- **What you should be able to do after this module**

  1. Understand what a model is
  2. Describe the impedance of a Resistor, Capacitor, and Inductor



## Interconnect Modeling

- **Interconnect Modeling**

- All interconnect can be described using the fundamentals of electromagnetic wave propagation given by Maxwell's equations.

- However, it is impractical to use Maxwell's equations in real designs due to the time associated with the solutions.

- Instead, we try to *Model* the performance of the interconnect using our basic circuit elements.

- We use the word *Model* to describe the schematic of a circuit that mimics the electrical behavior of a physical structure.

- A model not only gives us a gut feel, but it can be simulated with SPICE.

- Since simulators only operate on ideal components, we need to construct our equivalent circuit model using

i.e., Resistors, Capacitors, Inductors, and Transmission Lines



## Interconnect Modeling

- **Interconnect Modeling**

- The accuracy of a model can be increased by using more circuit elements.

- However, more circuit elements cause the simulation to run slower.

- As a result, we want a *Model* that is:

1) Accurate enough for our application
2) Not overly complex resulting in longer-than-necessary simulation times

- A model is also be accurate over a finite frequency range. We call this the:

*"Bandwidth of the Model"*

- The model may be accurate over its bandwidth, but may give inaccurate results above its BW.

- We need to make sure that our model has sufficient bandwidth to accurately predict the behavior for all spectral content in our system.

- Again, the more circuit elements in the model, the higher range of frequency that the model can cover.



## Interconnect Modeling

- **Interconnect Modeling**

- There are 4 ideal circuit elements that we use to describe a circuit's behavior:

| <u>Lumped Elements</u> | <u>Distributed Elements</u> |
|---|---|
| Resistor (R)<br>Capacitor (C)<br>Inductor (L) | Transmission Line (T) |

- "Lumped" means there is no propagation time through the element

- "Distributed" means there **IS** propagation time through the element



## Impedance

- **Impedance**

- In all cases, the most important electrical parameter of a system is its **Impedance**

- Impedance is ALWAYS defined as:

$$Z = \frac{V}{I}$$

- At first glance, this looks just like a resistor, but Impedance is the generic expression that includes time & frequency dependence.



## Impedance (R)

- **Z of a Resistor**

- Resistance is the Ratio of DC Voltage to DC Current

- The fundamental expression that describes the performance of a resistor is ***Ohm's Law***:

$$V = I \cdot R$$

- substituting this into the definition for Impedance, we can find $Z_R$

$$Z_R = \frac{V}{I} = \frac{I \cdot R}{I} = R$$
$$Z_R = R$$

- this is obvious, but one important takeaway is that an ideal resistor element has a fixed impedance that is **not dependant on frequency**

## Impedance (C)

- **Z of a Capacitor**

  - Capacitance is the Ratio of Charge stored between two conducting nodes to the voltage across the same nodes:

$$C = \frac{Q}{V}$$

  - Large Capacitance means that more charge can be stored with less voltage.

  - Physically, a capacitor is made of two plates separated by an insulating (dielectric) material.

    - Since the two plates are isolated, there is **NO DC Current**

    - if we inject charge suddenly on one of the capacitor nodes, the dielectric will polarize and cause charge to move on the opposite node. This instantaneous charge movement is **AC** or **transient current.**



## Impedance (C)

- **Z of a Capacitor**

  - If we look at the definition of Current, we can form a relationship for the behavior of a capacitor:

$$I = \frac{dQ}{dt}$$

  - Let's rearrange our capacitor definition and then differentiate with respect to time:

$$C = \frac{Q}{V}$$

$$Q = CV$$

$$\frac{dQ}{dt} = C\frac{V}{dt}$$

$$i_c = C\frac{V}{dt}$$

  - This is the fundamental expression that describes the behavior of a capacitor.

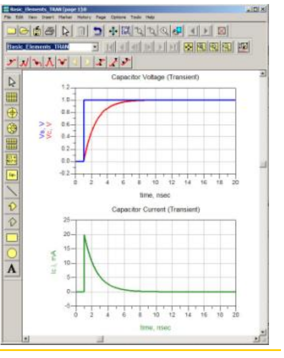


## Impedance (C)

- **Z of a Capacitor**

  - This expression says that voltage cannot change instantaneously but current can.

  - A small change in voltage will result in a large change in current.






## Impedance (C)

- **Z of a Capacitor**

  Time Domain

  - if we look at the impedance of a capacitor in the time domain, we first remember that impedance is always V/I

$$Z = \frac{V}{I} = \frac{V}{C\frac{dV}{dt}}$$

  - this expression that says:

    1) when dV/dt is small (or DC), the capacitor impedance is HIGH

    2) when dV/dt is large (or High Frequency), the capacitor impedance is LOW

  - we conceptually say:

    1) At DC, a capacitor looks like an **OPEN**.
    2) At High Frequency, a capacitor looks like a **SHORT**.



## Impedance (C)

- **Z of a Capacitor**

  Time Domain

  - let's represent our voltage in the time domain using sine waves:

$$V = V_0 \sin(\omega t)$$

  - now let's derive the current of a capacitor using this voltage:

$$i_c = C\frac{dV}{dt} = C\frac{d(V_0 \sin(\omega t))}{dt}$$

$$i_c = \omega \cdot C \cdot V_0 \cos(\omega t)$$

  - Now plugging into our Impedance expression for capacitance, we get:

$$Z_C = \frac{V}{I} = \frac{V_0 \sin(\omega t)}{\omega \cdot C \cdot V_0 \cos(\omega t)}$$

$$Z_C = \frac{1}{\omega \cdot C} \cdot \frac{\sin(\omega t)}{\cos(\omega t)}$$



## Impedance (C)

- **Z of a Capacitor**

  Time Domain

$$Z_C = \frac{1}{\omega \cdot C} \cdot \frac{\sin(\omega t)}{\cos(\omega t)}$$

  - This expression tells us two important things:

    1) The magnitude of the Impedance is: $|Z_C| = \frac{1}{\omega \cdot C}$

    2) The phase of the Impedance is: $ang(Z_C) = -90^o$

  - we say that the Voltage LAGS behind the Current in a capacitor by 90º, or "ICE"

## Impedance (C)

- **Z of a Capacitor**

Frequency Domain

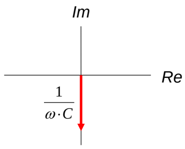
- In the frequency domain, we only have sine waves with Magnitude, Frequency, and Phase.

- We use a complex plane to represent the magnitude and phase with one complex quantity.

- On the complex plane, we represent a -90º phase using a -j

- Remember on the complex plane:

$$Z = a + jb$$
$$|Z| = \sqrt{a^2 + b^2}$$
$$ang(Z) = \tan^{-1}\left(\frac{a}{b}\right)$$





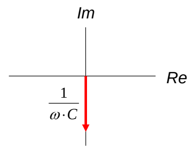

## Impedance (C)

- **Z of a Capacitor**

Frequency Domain

- The impedance of the capacitor can be expressed as:

$$Z_C = -j \cdot \left(\frac{1}{\omega \cdot C}\right)$$
$$Z_C = \frac{1}{j\omega \cdot C} = \frac{1}{C \cdot s}$$




- Note that Impedance is a *complex quantity* and we define the *Complex Frequency* as s=jω

- We call the Real part of Impedance **Resistance** and the Imaginary part ***Reactance***.

- Since a capacitor only has an imaginary component, we can say that the Reactance is equal to the Impedance.

$$X_C = \frac{1}{C \cdot s} = Z_C$$



## Impedance (C)

- **Z of a Capacitor**

Frequency Domain

- We typically get the most information from the magnitude of the impedance.

$$|Z_C| = \frac{1}{\omega \cdot C} = \frac{1}{2\pi \cdot f \cdot C}$$

- Note this shows an inverse relationship between Impedance and Frequency

- This verifies what we saw in the Time Domain:

1) At DC, a capacitor looks like an **OPEN**.
2) At High Frequency, a capacitor looks like a **SHORT**.

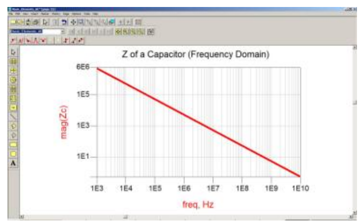


## Impedance (C)

- **Z of a Capacitor**

Frequency Domain

$$|Z_C| = \frac{1}{2\pi \cdot f \cdot C}$$






## Impedance (L)

- **Z of an Inductor**

- Inductance is the ratio of Magnetic Flux to Current

$$L = \frac{\Phi}{I}$$

- Magnetic Flux is the number of B-field Lines around the conductor (units are Webers, Wb)

- Large Inductance means that more magnetic fields can be stored with less current.

- Physically, an inductor is a structure or material that can temporarily hold Magnetic field lines ex) a coil, or Ferroelectric material



## Impedance (L)

- **Z of an Inductor**

- Faraday's Law of Induction states that the voltage induced from an inductor is:

$$V = \frac{d\Phi}{dt}$$

- Let's rearrange our inductance definition and then differentiate with respect to time:

$$L = \frac{\Phi}{I}$$
$$\Phi = L \cdot I$$
$$\frac{d\Phi}{dt} = L\frac{dI}{dt}$$
$$v_L = L\frac{dI}{dt}$$

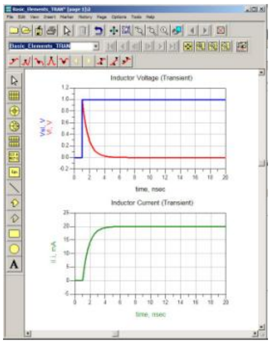
- This is the fundamental expression that describes the behavior of an inductor.

## Impedance (L)

- **Z of an Inductor**

  - This expression says that current cannot change instantaneously but voltage can.

  - A small change in current will result in a large change in voltage.






## Impedance (L)

- **Z of an Inductor**

  Time Domain

  - if we look at the impedance of an inductor in the time domain, we first remember that impedance is always V/I

$$Z = \frac{V}{I} = \frac{L \cdot \frac{dI}{dt}}{I}$$

  - this expression that says:

    1) when dI/dt is small (or DC), the inductor impedance is LOW

    2) when dI/dt is large (or High Frequency), the inductor impedance is HIGH

  - we conceptually say:

    1) At DC, an inductor looks like a **SHORT**.
    2) At High Frequency, an inductor looks like an **OPEN**.



## Impedance (L)

- **Z of an Inductor**

  Time Domain

  - let's represent our current in the time domain using sine waves:

$$I = I_0 \sin(\omega t)$$

  - now let's derive the voltage of an inductor using this current:

$$v_L = L\frac{dI}{dt} = L\frac{d(I_0 \sin(\omega t))}{dt}$$
$$v_L = \omega \cdot L \cdot I_0 \cos(\omega t)$$

  - Now plugging into our Impedance expression for inductance, we get:

$$Z_L = \frac{V}{I} = \frac{\omega \cdot L \cdot I_0 \cos(\omega t)}{I_0 \sin(\omega t)}$$
$$Z_L = \omega \cdot L \cdot \frac{\cos(\omega t)}{\sin(\omega t)}$$



## Impedance (L)

- **Z of an Inductor**

  Time Domain

$$Z_L = \omega \cdot L \cdot \frac{\cos(\omega t)}{\sin(\omega t)}$$

  - This expression tells us two important things:

    1) The magnitude of the Impedance is: $|Z_L| = \omega \cdot L$

    2) The phase of the Impedance is: $ang(Z_L) = +90^o$

  - we say that the Voltage LEADS the Current in an inductor by 90º, or "ELI"



## Impedance (L)

- **Z of an Inductor**

  Frequency Domain

  - In the frequency domain, we only have sine waves with Magnitude, Frequency, and Phase.

  - We use a complex plane to represent the magnitude and phase with one complex quantity.

  - On the complex plane, we represent a +90º phase using a +j

  - Remember on the complex plane:

$$Z = a + jb$$
$$|Z| = \sqrt{a^2 + b^2}$$
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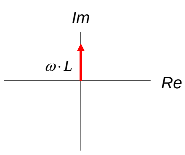




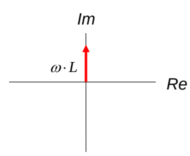

## Impedance (L)

- **Z of an Inductor**

  Frequency Domain

  - The impedance of the inductor can be expressed as:

$$Z_L = j \cdot \omega \cdot L = s \cdot L$$




  - Again, we call the Real part of Impedance **Resistance** and the Imaginary part ***Reactance***.

  - Since an inductor only has an imaginary component, we can say that the Reactance is equal to the Impedance.

$$X_L = s \cdot L = Z_L$$

## Impedance (L)

- **Z of an Inductor**

  Frequency Domain

  - In the Frequency Domain, the magnitude of the impedance is:

$$|Z_L| = \omega \cdot L = 2\pi \cdot f \cdot L$$

  - Note this shows a linear relationship between Impedance and Frequency

  - This verifies what we saw in the Time Domain:

    1) At DC, an inductor looks like a **SHORT**.
    2) At High Frequency, an inductor looks like an **OPEN**.



## Impedance (L)

- **Z of an Inductor**

  Frequency Domain

$$|Z_L| = \omega \cdot L = 2\pi \cdot f \cdot L$$