EE 477 Digital Signal Processing 7b *z*-Transforms

Inverse and Deconvolution

- We have seen that two systems in cascade can be combined into a single system by multiplying H₁(z) and H₂(z).
- We can also take a system function H(z) and *factor* it into two or more low-order systems.
- Question: can we *divide* the system output by the system function ("deconvolve") and recover the input?

$$Y(z) = H_1(z)X(z); \quad Y(z)H_2(z) = X(z)$$

 $Y(z) = H_1(z)H_2(z)Y(z) \Longrightarrow H_1(z)H_2(z) = 1?$

Inverse and Deconvolve, cont.

- If we can find $H_2(z)$, it is called the *inverse* of $H_1(z)$.
- NOTE that H₂(z) will not be FIR if H₁(z) is FIR.
- H₂(z) may represent a non-causal and/or unstable system even if H₁(z) is causal and stable.

Relating H(z) and H($e^{j\omega}$)

• NOTE CAREFULLY: z-transform and frequency response formulae are of identical form.

$$H(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} \quad H(z) = \sum_{k=0}^{M} b_k z^{-k}$$

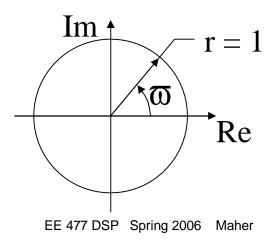
• If we evaluate H(z) for $z=e^{j\omega}$, it is clear:

$$H(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\omega}}$$

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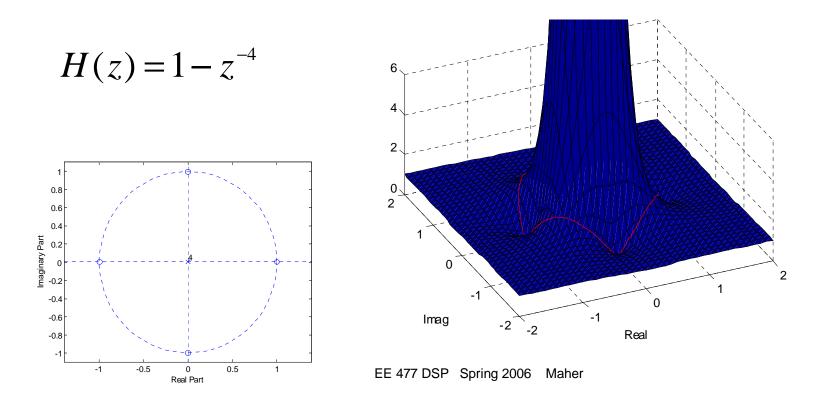
Properties of z=e^j™

- This defines a *circle* in the z-plane with radius=1: referred to as the *unit circle*



Visualizing Frequency Response

• We can observe z-transform along the unit circle to reveal the frequency response.



Poles and Zeros

- A *pole* in the z-domain is a value of *z* that "pushes up" the magnitude like a tent pole.
- A *zero* in the z-domain is a value of *z* that "pins down" the magnitude like a stake or tack.
- The pole and zero locations control the magnitude everywhere, *including along the unit circle*.

FIR Systems

- FIR systems contain only finite zeros.
 Poles are located at zero (and perhaps infinity).
- FIR filter design requires a careful choice of zero locations.
- Stop band has zeros on the unit circle.
- Pass band has zeros off the unit circle.

Matlab FIR Filter Design

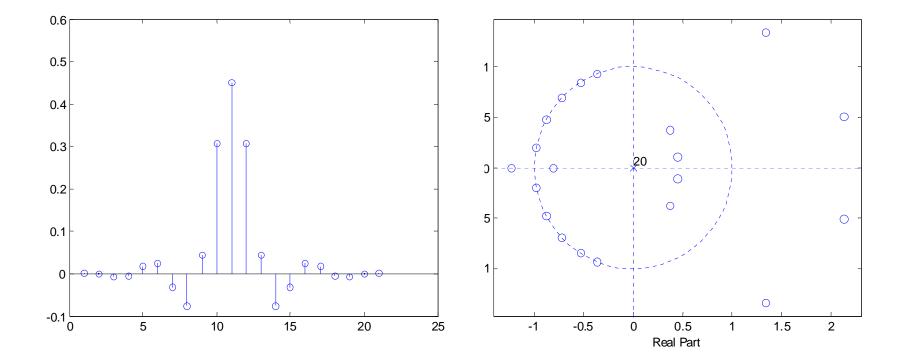
- Matlab provides several FIR filter design tools, including: fir1, fir2, and remez
- Matlab GUI: Filter Design and Analysis Tool (FDATool)
- Usually specify passband ripple, stopband attenuation, band edges, filter order, and $\rm f_{s}$

Design Example

| 🔰 Filter Design & Analysis Tool - [| untitled.fda *] | | |
|--|------------------------------------|-----------------|--|
| <u>Eile E</u> dit <u>A</u> nalysis <u>W</u> indow <u>H</u> elp | | | |
| ର 🖉 🔚 🚭 🚺 ର 🔤 🗩 | 🔁 🛼 📐 😡 🔂 🗯 🏦 🖵 🤘 | B 😡 🕅 | |
| Current Filter Information | Magnitude and Phase Response | 3 | |
| | Magnitude (dB) and Phase Responses | | |
| | 50 | | 0 |
| | 10 | | |
| Structure: Direct form FIR | | | (Se |
| Order: 20 | (gp) -30 | | -480 (See aligned and a see al |
| Sections: 1 | | h | -720 8 |
| Stable: Yes | Wag | | |
| Source: Designed | -110 | | |
| | -150 | | -1200 |
| | -150 | 10 15 | 20 |
| | | Frequency (kHz) | |
| | | | Manufada Osasili selises |
| Filter Type | | Units: Hz | Magnitude Specifications |
| C Highpass | Specify order: 20 | | |
| C Bandpass | O Minimum order | Fs: 48000 | The attenuation at cutoff |
| C Bandstop | Options | Fc: 10800 | frequencies is fixed at 6 dB |
| C Differentiator | Scale passband | | (half the passband gain) |
| Design Method | Window: Kaiser 🔻 | 1 | (|
| | Function name: | | |
| C IIR Butterworth | Beta 5 | | |
| FIR Window | Update | | |
| | Design | Filter | |
| La | Lesign | | |
| omputing Response done. | | | |

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Design Example (cont.)



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Symmetry and Linear Phase

- FIR systems with symmetric coefficients (b_k=b_{M-k}) have frequency responses with linear phase.
- Show this by grouping z-transform terms, for example:

 $H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_2 z^{-4} + b_1 z^{-5} + b_0 z^{-6}$ = $z^{-3} [b_0 (z^3 + z^{-3}) + b_1 (z^2 + z^{-2}) + b_2 (z^1 + z^{-1}) + b_3]$

Linear Phase (cont.)

• Now evaluate H(z) on unit circle:

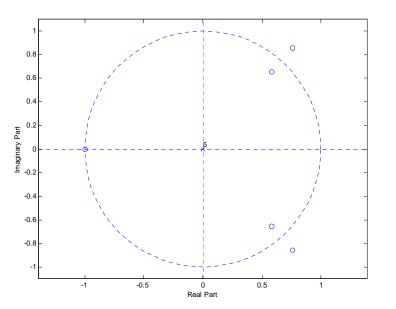
• Example if M is odd:

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + b_2 z^{-3} + b_1 z^{-4} + b_0 z^{-5}$$

= $z^{-2.5} [b_0 (z^{2.5} + z^{-2.5}) + b_1 (z^{1.5} + z^{-1.5}) + b_2 (z^{0.5} + z^{-0.5})]$

Zero Symmetry

For an FIR linear phase system (implies coefficient symmetry), the zeros will have a specific pattern. For each z₀, there will be:



$$\left\{z_{0}, z_{0}^{*}, \frac{1}{z_{0}}, \frac{1}{z_{0}}\right\}$$

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