EE577 Mid-term Exam #1 (SP09)

February 23, 2009 Name: ____________________

Open book and notes. No consultants. *Four Problems, Three Pages*

Part I (10 pts.) A causal LTI system is defined by the z-transform

$$H(z) = \frac{7}{\left(1 - \frac{2}{3}z^{-1}\right)\left(1 + \frac{3}{2}z^{-1}\right)}.$$

(a) What are the two poles and the two zeros of $H(z)$?

$P_1: \frac{2}{3}$ $P_2: \frac{3}{2}$

$z_1 = z_2 = 0$

(b) Specify the region of convergence (ROC) for $H(z)$.

ROC: $|z| > \frac{3}{2}$

(since right sided)

(c) Is the system stable?

no: ROC outside unit circle

Part II (10 pts.) A real discrete-time signal $x[n]$ is defined

$$x[n] = \left(\frac{1}{4}\right)^n u[n] + 2u[n-2]$$

Determine $X(z)$ and its region of convergence.

From table:

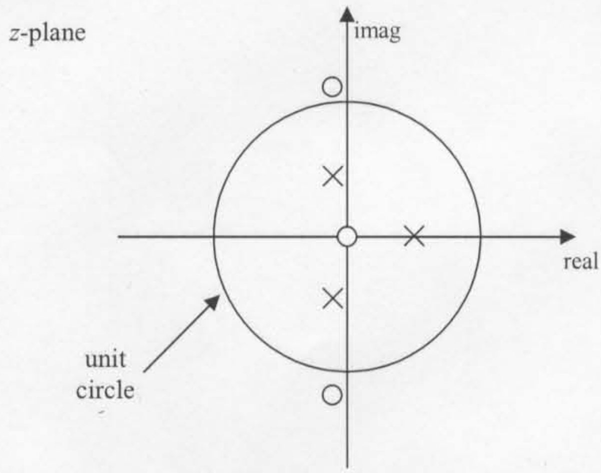
$\left(\frac{1}{4}\right)^n u[n] \longleftrightarrow \frac{1}{1 - \frac{1}{4}z^{-1}}$ ROC: $|z| > \frac{1}{4}$ pole at $\frac{1}{4}$ zero at 0

$2u[n-2] \Longleftrightarrow \frac{2z^{-2}}{1 - z^{-1}}$ ROC: $|z| > 1$ poles at 0 & 1, 2 zeroes at ∞

overall ROC is intersection: $|z| > 1$

Part III (10 pts.)

Determine a simple discrete-time difference equation (right sided) with real coefficients that has a system function described by the following pole-zero diagram.



zeros: $-0.1 \pm j1.1$; and 0
poles: $-0.1 \pm j0.4$, and 0.5

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z(z^2 + 0.2z + 1.22)}{(z-0.5)(z^2 + 0.2z + 0.17)}$$

$$= \frac{1 + 0.2z^{-1} + 1.22z^{-2}}{(1-0.5z^{-1})(1 + 0.2z^{-1} + 0.17z^{-2})}$$

Write as

$$\frac{A}{1-0.5z^{-1}} + \frac{B + Cz^{-1}}{1 + 0.2z^{-1} + 0.17z^{-2}}$$

Solve for A, B, C :

$A = 3.0192 \quad B = -2.0192 \quad C = -1.4135$

Recognize first term as

$A(0.5)^n u[n]$

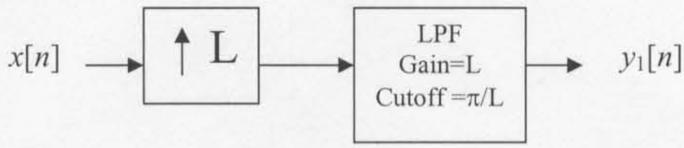
Recognize second term as a sum of damped sine + cosine terms

(would solve for coefficients) $\quad \{\alpha \cdot r^n \cos \omega_0 n + \beta \cdot r^n \sin \omega_0 n\} u[n]$

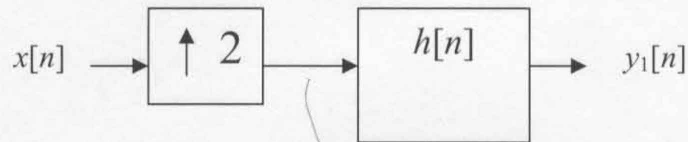
Solving:

$$\underbrace{3.0192(0.5)^n u[n]}_{\text{first term}} \; \underbrace{- 2.0192(0.4123)^n \cos(1.8157n)\, u[n] + 12.2369(0.4123)^n \sin(1.8157n)\, u[n]}_{\text{second term}}$$

Part IV (20 pts.) Recall that the general form for an upsampling operation is:

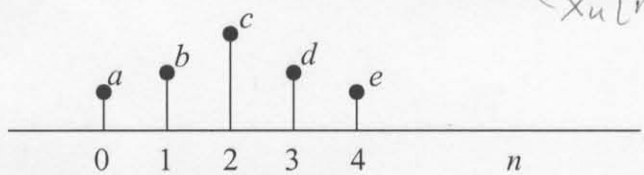


In a particular implementation, we want to perform the following:



where $h[n]$ is a 5-point FIR filter defined by:

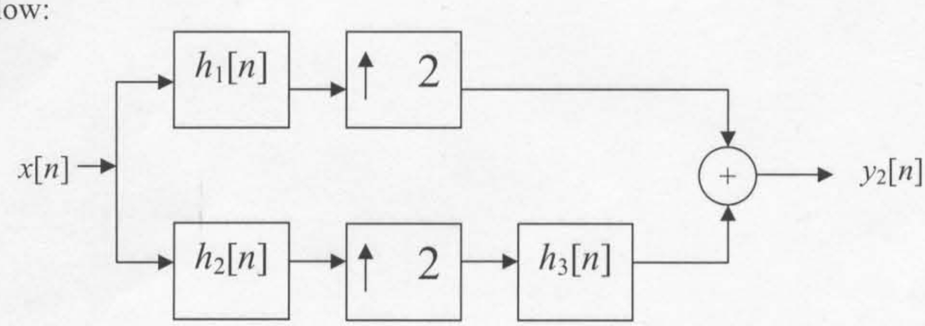
$x_u[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-2k]$



(a) Write an expression for $y_1[n]$ in terms of $x[n]$ and $h[n]$.

$$y_1[n] = x_u[n] * h[n]$$
$$= \sum_{q=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-2k] \right) \cdot h[n-q]$$

Now, you need to implement the upsampling system with the efficient polyphase implementation below:



(b) If the three unit sample responses $h_1[n]$, $h_2[n]$, and $h_3[n]$ are all restricted to be zero outside the range $0 \le n \le 2$, <u>determine and explain</u> your choices for $h_1[n]$, $h_2[n]$, and $h_3[n]$ so that $y_2[n]$ is identical to $y_1[n]$

$$h_1[n] = a\,\delta[n] + c\,\delta[n-1] + d\,\delta[n-2]$$
$$h_2[n] = b\,\delta[n] + d\,\delta[n-1]$$
$$h_3[n] = \delta[n-1]$$