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## Lab 5: Spectrograms: Harmonic Lines & Chirp Aliasing

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**Pre-Lab:** Read the Pre-Lab and do all the exercises in the Pre-Lab section *prior to attending lab*.

**Verification:** The Exercise section of each lab should be completed **during your assigned Lab time** and the steps marked *Instructor Verification* signed off **during the lab time**.

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### 1 Pre-Lab

#### 1.1 Objective

The objective of this lab is to study further the spectral content of signals analyzed via the *spectrogram*. There are several specific steps that will be considered in this lab:

1. Synthesize a linear-FM chirp with a MATLAB M-file, and display its spectrogram. Choose the chirp parameters so that aliasing will happen.
2. Synthesize a periodic triangle wave with a MATLAB M-file, and display its spectrogram. Relate the harmonic line spectrum to the fundamental period of the triangle wave.
3. Compare spectrograms using different scales for amplitude: decibels (dB) for amplitude versus linear amplitude.
4. Examine details of the harmonic lines in the dB spectrogram of the triangle wave.
5. *Spectrogram:* make a spectrogram of your voice signal, and relate the harmonic line spectrum to your previous measurement of pitch period.

#### 1.2 Chirp, or Linearly Swept Frequency

A linear-FM *chirp* signal is a sinusoid whose instantaneous frequency changes linearly from a starting value to an ending one. The formula for such a signal can be defined by creating a complex exponential signal with a quadratic angle function  $\psi(t)$ . Mathematically, we define  $\psi(t)$  as

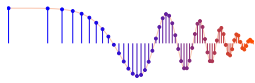
$$\psi(t) = 2\pi\mu t^2 + 2\pi f_0 t + \varphi \quad (1)$$

The derivative of  $\psi(t)$  yields an instantaneous *cyclic* frequency that changes *linearly* versus time.

$$f_i(t) = 2\mu t + f_0 \quad (\text{hertz}) \quad (2)$$

The slope of  $f_i(t)$  is equal to  $2\mu$  and its  $t = 0$  intercept is  $f_0$ . The frequency variation in (2) produced by the time-varying angle function is called *frequency modulation*, so these signals are called FM signals. Finally, since the linear variation of the frequency (2) can produce an audible sound similar to a siren or a bird chirp, linear-FM signals are also called *chirps*.

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If the signal starts at time  $t = t_1$  s with a frequency of  $f_1$  Hz, and ends at time  $t = t_2$  s with a frequency of  $f_2$  Hz, then the slope of the line in (2) will be

$$\text{SLOPE} = 2\mu = \frac{f_2 - f_1}{t_2 - t_1} \quad (3)$$

Note that if the signal starts at time  $t = 0$  s, then  $f_1 = f_0$  is also the starting frequency. Otherwise,  $f_0 = ?$

### 1.2.1 MATLAB Synthesis of Chirp Signals

In MATLAB signals can only be synthesized by evaluating the signal's defining formula at discrete instants of time. These are called *samples* of the signal. For the chirp we use the following:

$$x(t_n) = A \cos(2\pi\mu t_n^2 + 2\pi f_0 t_n + \varphi)$$

where  $t_n$  is the  $n^{\text{th}}$  time sample. The following MATLAB code will synthesize a linear-FM chirp:

```
fSamp = 8000;    %-Number of time samples per second
dt = 1/fSamp;
tStart = 0;
tStop = 1.5;
tt = tStart:dt:tStop;
mu = 600;
fzero = 400;
phi = 2*pi*rand;    %-- random phase
%
%%  psi = ????.    <===== FILL IN THE CODE HERE
%
cc = real( 7.7*exp(j*psi) );
%  soundsc( cc, fSamp );    %-- uncomment to hear the sound
plotspec( cc+j*1e-12, fSamp, 256 ), colorbar, grid on    %-- with negative frequencies
```

### 1.3 Triangle Wave

Write a simple MATLAB script that will generate a periodic triangle wave once the period ( $T$ ) is given. The DC level of the triangle wave should be zero, and the peak amplitude should be equal to 0.5, i.e., the signal extremes should be  $\pm 0.5$ . Here is a MATLAB one-liner that can form the basis of this script:

```
tt=0:(1/fs):tStop;qq=rem(tt,T);xx=Amp*(abs(qq-(0.5*T))-0.25*T);
```

The values of  $fs$ ,  $tStop$ ,  $T$ ,  $Amp$  will have to be parameters.

- Test with  $fs=1000$ ,  $tStop=1$ ,  $T=0.2$ ,  $Amp=9$  by making a plot.
- Find  $Amp$  to get positive peaks at  $\frac{1}{2}$  and negative peaks at  $-\frac{1}{2}$ .

### 1.4 Decibels

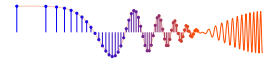
The common log scale used in engineering is decibels (dB), which is defined as  $20\log_{10}(A)$  where  $A$  is amplitude. The built-in MATLAB spectrogram M-file uses a dB scale for amplitude when displaying its spectrogram image. The decibel has two notable features:

1. *Ratios become Differences:* On a dB scale, a numbers are represented with logarithms, so the ratio  $P/Q$  becomes

$$20\log_{10}(P/Q) = 20\log_{10}(P) - 20\log_{10}(Q)$$

If  $A_2 = (1/10)A_1$  then  $A_2$  is 20 dB lower than  $A_1$ , because with logs, we get

$$20\log_{10}(A_2) = 20\log_{10}((1/10)A_1) = 20\log_{10}(A_1) + 20\log_{10}(0.1) = 20\log_{10}(A_1) - 20\text{dB}.$$



Since ratios become differences, dB is most often used to compare the relative size of values.

- The dB range must be restricted because  $20 \log_{10}(0) = -\infty$ . If we want to map the linear amplitude range  $[0, 1]$  into dB, we must define a minimum dB level. Since  $20 \log_{10}(1) = 0$  dB is the maximum, other dB values for  $[0, 1]$  will be negative. The minimum dB level will chop off the bottom of  $[0, 1]$  and make it equal to  $[\epsilon, 1]$  where  $\epsilon$  is very small. For example, a dB range of 80 dB would define the minimum to be  $-80$  dB, so  $\epsilon = 10^{-80/20} = 10^{-4} = 0.0001$ .

- Express the numbers 0.1, 1, 2, 5, 10 and 100 in dB.
- Convert  $-6$  dB,  $-60$  dB, and  $-80$  dB to numbers.

## 1.5 Fourier Series of a Triangle Wave

The periodic triangular wave has a known Fourier Series. After consulting the *SP-First* text, we can write:

$$a_k = \begin{cases} \frac{-2}{\pi^2 k^2} & \text{for } k \text{ odd} \\ 0 & \text{for } k \text{ even} \end{cases} \quad (4)$$

for a *zero-mean* triangle wave, i.e., after dropping the DC term  $a_0$ , and whose maximum amplitude is 0.5,

- Evaluate the coefficients for  $k = 1, 3, 5$  and 15. Then compute the ratios  $a_3/a_1$ ,  $a_5/a_1$  and  $a_{15}/a_1$ .

*Comment:* Here's a general question: **are the Fourier Series coefficients independent of time scaling ?** First of all,  $y(t) = x(bt)$  is scaling. For example, a plot of  $x(3t)$  will be squeezed along the horizontal axis by a factor of  $b = 3$ .

What are the Fourier coefficients of  $y(t) = x(bt)$  in terms of the Fourier coefficients of  $x(t)$  ? If  $x(t)$  has a period equal to  $T$ , then the period of  $y(t)$  is  $T/b$  because  $x(t)$  is squeezed by  $b$ . Thus, the fundamental frequency of  $y(t)$  is

$$\frac{2\pi}{(T/b)} = b \left( \frac{2\pi}{T} \right) \quad \text{rad/s}$$

that is, it is scaled by  $b$ .

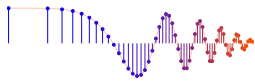
Now we write the Fourier Series integral for  $y(t)$

$$(1/(T/b)) \int_{-T/2b}^{T/2b} y(t) e^{-j((2\pi k)/(T/b))t} dt = (b/T) \int_{-T/2b}^{T/2b} x(bt) e^{-j(2\pi k/T)(bt)} dt$$

Make a change of variables:  $\lambda = bt$ , and with  $d\lambda = b(dt)$ , you get

$$b(1/T) \int_{-T/2}^{T/2} x(\lambda) e^{-j(2\pi k/T)\lambda} (1/b) d\lambda = (1/T) \int_{-T/2}^{T/2} x(\lambda) e^{-j(2\pi k/T)\lambda} d\lambda = a_k$$

The scaling factor  $b$  cancels to give the RHS. So, time scaling doesn't change the  $\{a_k\}$  coefficients. The Fourier Series coefficients of  $y(t)$  are exactly equal to the Fourier Series coefficients of  $x(t)$ .



## 2 Lab Exercise

For the lab exercise, you will synthesize some signals, and then study their frequency content by using the spectrogram. The objective is to learn more about the connection between the *time-domain* definition of the signal and its *frequency-domain* content.

For the instructor verification, you will have to demonstrate that you understand concepts in a given subsection by answering questions from your lab instructor (or TA).

### 2.1 Spectrogram for a Chirp that Aliases

Use the code provided in the pre-Lab section as a starting point in order to write a MATLAB script or function that will synthesize a “chirp” signal. Then use that M-file in this section.

- What happens when we make a signal that “chirps” up to a very high frequency, and the *instantaneous frequency goes past half the sampling rate*? Generate a chirp signal that starts at 1000 Hz when  $t = 0$  s, and chirps up to 11,000 Hz, at  $t = 4$  s. Use  $f_s = 4000$  Hz. Determine the parameters needed in (1).
- Generate the chirp signal in MATLAB and make a spectrogram with a short section length,  $L_{\text{SECT}}$ , to verify that you have the correct starting and ending frequencies.<sup>2</sup> For your chosen  $L_{\text{SECT}}$ , determine the section duration  $T_{\text{SECT}}$  in secs.
- Explain why the instantaneous frequency seen in the spectrogram is *goes up and down* between zero and  $f_s/2$ , i.e., it does not chirp up to 11,000 Hz. There are two effects that should be accounted for in your explanation.

*Note:* If possible listen to the signal to verify that the spectrogram is faithfully representing the audio signal that you hear.

**Instructor Verification** (separate page)

### 2.2 Spectrogram of Periodic Signal

A periodic signal is known to have a Fourier Series, which is usually described as a *harmonic line spectrum* because the only frequencies present in the spectrum are integer multiples of the fundamental frequency. With the spectrogram, it is easy to exhibit this harmonic line characteristic.

#### 2.2.1 Spectrogram of Periodic Triangle Wave

- Write a simple MATLAB script that will generate a periodic triangle wave once the period is given. The DC level of the triangle wave should be zero, and the peak amplitude should be equal to 0.5. Here is a MATLAB one-liner that can form the basis of this script:

```
tt=0:(1/fs):tStop;qq=rem(tt,T);xx=Amp*(abs(qq-(0.5*T))-0.25*T);
```

The values of  $f_s$ ,  $t_{\text{Stop}}$ ,  $T$ ,  $Amp$  will have to be determined.

- Generate a triangle wave with a period of 10 msec, using a sampling rate of  $f_s = 10000$  Hz. The duration should be 3 secs. Then make a plot of a short section of the signal consisting of 3–5 periods to verify that you have the correct time waveform.

<sup>2</sup>There is no single correct answer for  $L_{\text{SECT}}$ , but you should pick a value that makes a smooth plot and lets you easily see the changing nature of the instantaneous frequency.



- (c) Make a spectrogram with a long section duration.<sup>3</sup> It is important to pick a section duration that is equal to an integer number of periods of the periodic triangular waveform created in the previous part. Define  $T_{\text{SECT}}$  to get exactly 5 periods, and then determine the section length  $L_{\text{SECT}}$  (an integer) to be used in `plotspec`.
- (d) You should expect to see a “harmonic line spectrum” in the spectrogram. Since frequency is along the vertical axis, the harmonic lines will appear as horizontal lines in the spectrogram. Make a list of all the harmonic frequencies that you can see in the spectrogram.
- (e) Determine the fundamental frequency for the harmonic lines.
- (f) Measure the amplitudes of the first and third harmonic lines by using MATLAB’s *Data Cursor* after zooming in on those parts of the spectrogram image. Record the values for the amplitudes and compute the ratio.

**Instructor Verification** (separate page)

### 2.2.2 Decibels (dB): Seeing Small Values in the Spectrogram

The periodic triangular wave has a known Fourier Series (see Sect. 1.5)

$$a_k = \begin{cases} \frac{-2}{\pi^2 k^2} & \text{for } k \text{ odd} \\ 0 & \text{for } k \text{ even} \end{cases}$$

for a *zero-mean* triangle wave that varies between  $\pm 0.5$ . Thus, in the spectrogram we should see harmonic lines at  $f_0, 3f_0, 5f_0$ , etc. Furthermore, there should be an infinite number of harmonic frequency lines.

*Where did all the harmonics go?*

The answer is that the higher harmonics have amplitudes that are too small to be seen in a spectrogram that displays values with a *linear amplitude*. Instead, a logarithmic amplitude scale is needed.

Consult Section 2.2.2 in the Pre-Lab for a discussion of *decibels*, and then answer the following questions about decibels:

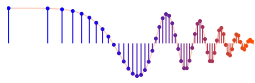
- (a) In the language of dB, a factor of two is “6 dB.” In other words, if  $B_2$  is 6 dB bigger than  $B_1$ , then it is twice as big (approximately). Explain why this statement is true.
- (b) The nonzero Fourier coefficients of the triangular wave are  $a_k = -2/(\pi^2 k^2)$ . Determine the dB difference between  $a_1$  and  $a_3$ . In other words,  $a_3$  is how many dB below  $a_1$ . Furthermore, explain why the dB difference depends *only on the k indices*.
- (c) Determine (in dB) how far  $a_{15}$  is below  $a_1$  for the periodic triangular wave.

**Instructor Verification** (separate page)

### 2.2.3 Spectrogram in dB

A variation of the *SP-First* function `plotspec` has been written to incorporate the dB amplitude scale. This new function is called `plotspecDB`, and its calling template is shown below:

<sup>3</sup>A long section duration in the spectrogram yields what is called a *narrowband spectrogram* because it will provide excellent resolution of the frequency components of a signal.



```
function him = plotspecDB(xx,fsamp,Lsect,DBrange)
%PLOTSPECDB  plot a Spectrogram as an image
%           (display magnitude in decibels)
% usage:    him = plotspec(xx,fsamp,Lsect,DBrange)
%           him = handle to the image object
%           xx = input signal
%           fsamp = sampling rate
%           Lsect = section length (integer, power of 2 is a good choice)
%                   amount of data to Fourier analyze at one time
%           DBrange = defines the minimum dB value; max is always 0 dB
```

- Create a “dB-Spectrogram” for the 10-msec periodic triangular wave generated in Sect. 2.2.1. Use a dBRange equal to 80 dB. Notice that many more spectrum lines are now visible. List the frequencies of all the harmonic spectrum lines, or give a general formula.
- Generate a second triangle wave by changing the period to 20 msec. Then make the dB-Spectrogram of this 20-msec triangle wave, being careful to select the section duration as an integer number of periods. From the spectrogram, determine the fundamental frequency and also the frequency of the highest harmonic line. Also, determine the harmonic number for the highest frequency, e.g., the 17<sup>th</sup> or 31<sup>st</sup>, etc.
- For the 20-msec triangle wave, measure the amplitudes (in dB) of the first and third harmonic lines by using MATLAB’s *Data Cursor* after zooming in on those parts of the spectrogram image. Compare the dB difference to the ratio obtained in Section 2.2.1, part (f).

**Instructor Verification** (separate page)

- Change the period to 4 msec and make another dB-Spectrogram. Be careful to select the section duration as an integer number of periods. This period is shorter but the frequency separation of the harmonic lines is greater. Notice that this inverse relationship was also true when comparing the 20 msec case to the 10 msec case.

### 2.3 Lab-HW: dB Spectrogram of Your Voice [OPTIONAL]

Record a slowly spoken phrase "hat, cat, sat", and make a dB spectrogram with a long section length. Since the vowel region of the recording is a signal that is quasi-periodic, the spectrogram should exhibit a harmonic line characteristic during that time. Try different section lengths until the harmonic lines are easy to see in the spectrogram.

- Turn in a print out of the spectrogram. Annotate the spectrogram to show the vowel region, as well as parameter values and measurements such as:
  - The values of  $T_{\text{SECT}}$  and  $L_{\text{SECT}}$  used.
  - The measured frequency separation of the harmonic lines that are seen in the vowel region.
- From the frequency separation, compute the fundamental frequency in the vowel region. Also determine the fundamental period for this fundamental frequency.
- Recall the value of the pitch period (in secs) measured in Lab #1. Compare the fundamental period determined in the previous part.
- Comment on similarities and differences between the two period measurements.



## Lab: Spectrograms of Harmonic Lines & Chirp Aliasing

### INSTRUCTOR VERIFICATION SHEET

Turn in this page with your lab memo report.

Name: \_\_\_\_\_

Date: \_\_\_\_\_

#### Part 2.1 Chirp Aliasing

(a) Chirp formula:  $\cos(\psi(t)) =$

(b)  $L_{\text{SECT}} =$  \_\_\_\_\_ and  $T_{\text{SECT}} =$  \_\_\_\_\_

(c) Write explanation of “ups and downs” in spectrogram:

Verified: \_\_\_\_\_

#### Part 2.2.1 Spectrogram of periodic triangle wave using a long section duration.

(b)  $T_{\text{SECT}} =$  \_\_\_\_\_ and  $L_{\text{SECT}} =$  \_\_\_\_\_

(d) List frequencies of visible harmonics =

(e) Fundamental Frequency =

(f)  $|a_1| =$  \_\_\_\_\_ and  $|a_3| =$  \_\_\_\_\_

Verified: \_\_\_\_\_

#### Part 2.2.2 Questions about decibels:

(a) Explain 6 dB.

(b) dB difference between  $|a_1|$  and  $|a_3| =$

(c) How far is  $|a_{15}|$  below  $|a_1|$  in dB?

Verified: \_\_\_\_\_

#### Part 2.2.3 dB-Spectrogram of periodic triangle wave using a long-duration section.

(a) List frequencies of harmonic lines =

(b) Fundamental Frequency =

(b) Highest Frequency =

(b) Harmonic Number of Highest Frequency =

(c) In dB,  $a_1 =$  \_\_\_\_\_ and  $a_3 =$  \_\_\_\_\_

Verified: \_\_\_\_\_