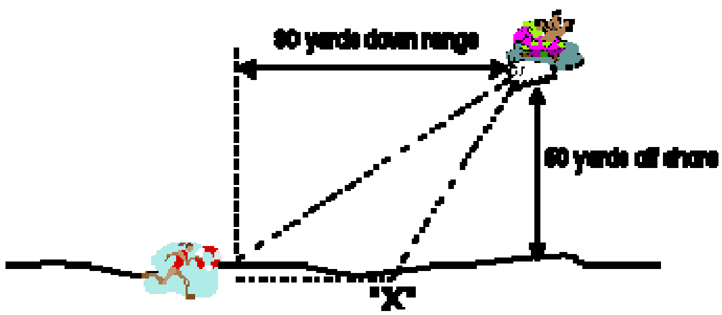
ENGR 125CS

Lifeguard Problem (FL07)

A lifeguard needs to reach a struggling swimmer as quickly as possible. No boat is available.
The struggling swimmer is 80 yards down the beach and 50 yards off shore, relative to the lifeguard.
The lifeguard can run along the beach at a speed of ***4 yards per second****, and she can swim through the water at a speed of* ***1 yard per second****.*



In order to reach the swimmer in as short a time as possible, how far along the shore should she run (point "X") before jumping in the water and swimming? What is the minimum time required?

Analytical solution:

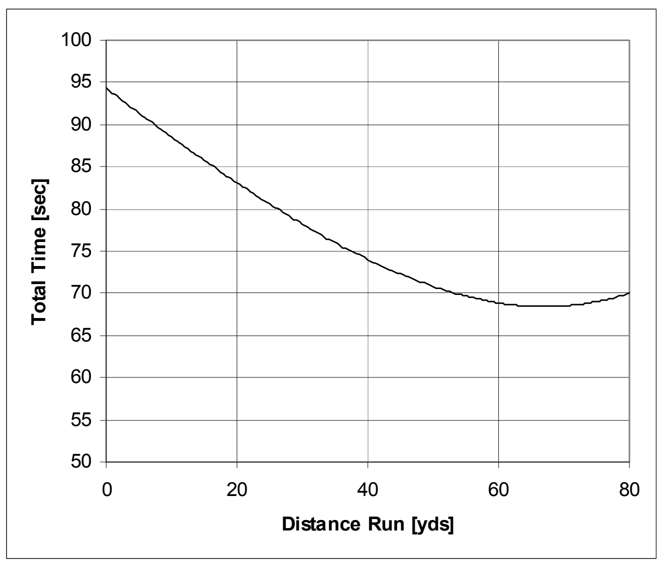
Total time = time running + time swimming
time running = distance run divided by run rate
time swimming = distance swum divided by rate

distance running = X

distance swimming = $\sqrt{50^2 + (80 - X)^2}$

so

Total time = $\dfrac{X}{4} + \dfrac{\sqrt{50^2 + (80 - X)^2}}{1}$



To find minimum time, take the derivative of the Total time expression with respect to X, set equal to zero, and solve for X.

$$\frac{d}{dX}(\text{Total time}) = \frac{1}{4} + \frac{1}{2\sqrt{8900 - 160X + X^2}} \cdot (2X - 160)$$

Solve for principal root:
X = 80-10/3*15^(1/2) = 67.09 yards (distance 'X' corresponding to minimum time)

Placing this result back into original Total time expression gives
Total time = 68.4 seco nds (minimum time to reach destination)